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Stability of a Rigid Rotor Supported on Oil-Film Journal Bearings Under Dynamic Load

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STABILITY OF A RIGID ROTOR SUPPORTED ON OIL-FILM JOURNAL BEARINGS UNDER DYNAMIC LOAD

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SUMMARY

Most published work relating to dynamically loaded journal bearings are directed to determining the minimum film thickness from the predicted journal trajectories. These do not give any information about the subsynchronous whirl stability of journal bearing systems since they do not consider the equations of motion. It is, however, necessary to know whether the bearing system operation is stable or not under such an operating condition.

The purpose of the present paper is to analyze the stability characteristics of the system. A linearized perturbation theory about the equilibrium point can predict the threshold of stability; however it does not indicate postwhirl orbit detail. The linearized method may indicate that a bearing is unstable for a given operating condition whereas the nonlinear analysis may indicate that it forms a stable limit cycle. For this reason, a nonlinear transient analysis of a rigid rotor supported on oil journal bearings under (1) a unidirectional constant load, (2) a unidirectional periodic load, and (3) variable rotating load are performed.

In this paper, the hydrodynamic forces are calculated after solving the time-dependent Reynolds equation by a finite difference method with a successive overrelaxation scheme. Using these forces, equations of motion are solved by the fourth-order Runge-Kutta method to predict the transient behavior of the rotor. With the aid of a high-speed digital computer and graphics, the journal trajectories are obtained for several different operating conditions.

INTRODUCTION

There are two principal approaches by means of which one can analyze the whirl instability of a rotor supported on fluid film bearings. These are: (1) linearized (perturbation) method and (2) nonlinear transient analysis. In

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the linearized method a small perturbation of the journal center about its equilibrium point is given. The stiffness and damping coefficients are determined after solving the basic differential equation. These coefficients are then used in the equations of motion to find the critical mass parameter and whirl ratio. The mass parameter, a function of rotor speed, is a measure of stability. The nonlinear transient analysis, on the other hand, gives the journal locus and from this one can know about the system stability.

Stability analysis of finite journal bearings by the linearized method has been given by Allaire (ref. 1), whereas Akers, Michaelson, and Cameron (ref. 2) studied the same bearing configurations using a nonlinear transient approach. In reference 2 it was shown that under certain operating conditions the journal motion was bounded and could form a limit cycle.

The aim of the present paper is to study theoretically the stability characteristics of finite oil journal bearings under dynamic load using a nonlinear transient method. A few papers (refs. 3 to 5) deal with the dynamically loaded bearings to predict the journal locus. As these do not consider the equations of motion for the prediction of the position of journal center, they cannot indicate whether the bearing system is stable or not. However, these are useful for estimating minimum film thickness of dynamically loaded bearings. The dynamical equations of motion are solved by fourth-order Runge-Kutta method to find eccentricity ratio and attitude angle and their derivatives for the next time step. These values are then introduced in the two-dimensional time-dependent Reynolds equation to find the hydrodynamic forces. Following the above approach a nonlinear transient analysis of a rigid rotor on oil journal bearings under (1) a unidirectional constant load, (2) a unidirectional periodic load, and (3) variable rotating load is performed. A number of trajectories have been obtained with the aid of a high-speed digital computer and graphics.

NOMENCLATURE

C	radial clearance
D	journal diameter
e	eccentricity
F_r, F_Θ,	hydrodynamic forces
$\bar{F}_r$, $\bar{F}_\Theta$	$\bar{F}_r = F_r C^2/\eta\omega R^3 L$, $\bar{F}_\Theta = F_\Theta C^2/\eta\omega R^3 L$ (dimensionless)
h, $\bar{h}$	film thickness, $\bar{h} = h/C$ (dimensionless)
M, $\bar{M}$	mass parameter, $\bar{M} = MC\omega^2/W_O$ dimensionless)
p, $\bar{p}$	film pressure, $\bar{p} = pC^2/\eta\omega R^3 L$
R	journal radius
T	dimensionless time, $T = \omega_p t$

t	time
x, z, θ, $\bar{z}$	coordinates, $\theta = x/R$, $\bar{z} = z/(L/2)$ (dimensionless)
W, $\bar{W}$	load, $W = WC^2/\eta\omega R^3 L$ (dimensionless)
W_o, $\bar{W}_o$	steady-state load, $\bar{W}_o = WC^2/\eta\omega R^3 L$ (dimensionless)
ε	eccentricity ratio, $\varepsilon = e/C$ (dimensionless)
η	absolute viscosity of oil
φ	attitude angle
Ψ_1, Ψ_2	angular coordinates at which film commences and cavitates, respectively, from minimum film thickness
θ_1, θ_2	$\theta_1 = \pi + \Psi_1$, $\theta_2 = \pi + \Psi_2$
Ω	whirl ratio, $\Omega = \omega_p/\omega$ (dimensionless)
ω	angular velocity of journal
ω_p	angular velocity of whirl

THEORY

The basic differential equation for pressure distribution in the bearing clearance under dynamic conditions can be written as (See fig. 1.)

$$\frac{\partial}{\partial x}\left(h^3 \frac{\partial P}{\partial x}\right) + \frac{\partial}{\partial z}\left(h^3 \frac{\partial P}{\partial z}\right) = 6\eta R\left(\omega - 2\frac{\partial\varphi}{\partial t}\right)\frac{\partial h}{\partial x} + 12\eta\frac{\partial h}{\partial t} \tag{1}$$

Equation (1) when nondimensionalized with the following substitutions: $\theta = x/R$, $\bar{z} = z/(L/2)$, $\bar{h} = h/C$, $\bar{p} = pC^2/\eta\omega R^2$, $T = \omega_p t$, and $\Omega = \omega_p/\omega$, will read as

$$\frac{\partial}{\partial\theta}\left(\bar{h}^3 \frac{\partial\bar{p}}{\partial\theta}\right) + \left(\frac{D}{L}\right)^2\left(\bar{h}^3 \frac{\partial^2\bar{p}}{\partial\bar{z}^2}\right) = 6\left(1 - 2\Omega\frac{\partial\varphi}{\partial T}\right)\frac{\partial\bar{h}}{\partial\theta} + 12\Omega\frac{\partial\bar{h}}{\partial T} \tag{2}$$

The film thickness (dimensionless) is

$$\bar{h} = 1 + \varepsilon\cos\theta \tag{3}$$

with the use of equation (3), equation (2) can be written as

$$\frac{\partial}{\partial\theta}\left(\bar{h}^3 \frac{\partial\bar{p}}{\partial\theta}\right) + \left(\frac{D}{L}\right)^2\left(\bar{h}^3 \frac{\partial^2\bar{p}}{\partial\bar{z}^2}\right) = 6(1 - 2\Omega\dot{\varphi})(-\varepsilon\sin\theta) + 12\Omega\dot{\varepsilon}\cos\theta \tag{4}$$

where $\dot{\varphi} = \partial\varphi/\partial T$ and $\dot{\varepsilon} = \partial\varepsilon/\partial T$. In this study we have used the Reynolds boundary conditions, which are given by:

$$p(\theta, \pm 1) = 0$$

$$\frac{\partial \bar{p}}{\partial z}(\theta, 0) = 0 \tag{5}$$

and

$$\left.\begin{aligned} \bar{p}(\theta, \bar{z}) &= 0 \\ \frac{\partial \bar{p}}{\partial \theta}(\theta, \bar{z}) &= 0 \end{aligned}\right\} \quad \theta_2 \leq \theta \leq \theta_1$$

where θ_1 and θ_2 are the angular coordinates at which the film commences and reforms, respectively.

Equation (4) is solved numerically for pressure by a finite difference method with a successive over relaxation scheme satisfying the above boundary conditions. Initially, ε and $\dot{\varphi}$ are set equal to zero to obtain the steady-state hydrodynamic forces $\bar{F}_r$ and $\bar{F}_\theta$.

The forces are computed from

$$\bar{F}_r = \int_0^1 \int_{\theta_1}^{\theta_2} \bar{p} \cos \theta \, d\theta \, d\bar{z}$$

and (6)

$$\bar{F}_r = \int_0^1 \int_{\theta_1}^{\theta_2} \bar{p} \sin \theta \, d\theta \, d\bar{z}$$

where

$$\bar{F}_r = \frac{F_r C^2}{\eta \omega R^3 L} \quad \text{and} \quad \bar{F}_\theta = \frac{F_\theta C^2}{\eta \omega R^3 L}$$

Releasing the journal from the steady-state position, one can compute ε, φ, $\dot{\varepsilon}$, and $\dot{\varphi}$ for the next time step by solving the following equations of motion:

$$MC\left[\frac{d^2\varepsilon}{dt^2} - \varepsilon\left(\frac{d\varphi}{dt}\right)^2\right] = F_r + W \cos \varphi \tag{7}$$

$$MC\left[\varepsilon \frac{d^2\varepsilon}{dt^2} + 2\left(\frac{d\varphi}{dt}\right)\left(\frac{d\varepsilon}{dt}\right)\right] = F_\theta - W \sin \varphi \tag{8}$$

The dimensionless form of equations (7) and (8) are:

$$\bar{M}\Omega^2\ddot{\varepsilon} - \bar{M}\Omega^2\varepsilon\dot{\varphi}^2 - \frac{\bar{F}_r}{\bar{W}_o} - \cos \varphi = 0 \tag{9}$$

$$\bar{M}\Omega^2\ddot{\varepsilon} + 2\bar{M}\Omega^2\dot{\varepsilon}\dot{\varphi} - \frac{\bar{F}_\theta}{\bar{W}_o} + \sin\varphi = 0 \tag{10}$$

where

$$\bar{M} = \frac{MC\omega^2}{W_o} \quad \text{and} \quad \bar{W}_o = \frac{W_o C^2}{\eta\omega R^3 L}$$

The steady-state load $\bar{W}_O$ is obtained by letting $\dot{\varepsilon}$ and $\dot{\varphi}$ equal to zero.

Equations (9) and (10) are second-order differential equations in ε and φ. These are solved by using a fourth-order Runge-Kutta method for constant values of Ω, $\bar{M}$, $\bar{F}_r$, and $\bar{F}_\theta$.

In the following section three types of load are considered.

A Unidirectional Constant Load

Assuming the initial conditions given in table I, the hydrodynamic forces under steady-state condition are found. Equations of motion are solved to obtain ε, $\dot{\varepsilon}$, φ, and $\dot{\varphi}$ for the subsequent time step. Now the new value of ε, $\dot{\varepsilon}$, and $\dot{\varphi}$ are introduced in equations (3) and (4) to determine the hydrodynamic forces. These forces along with the steady-state load, mass parameter and whirl ratio are utilized for the solution of equations (9) and (10). The process is repeated until we get a trajectory that describes the status of the system.

A Unidirectional Periodic Load

The applied load was assumed to be

$$\bar{W} = \bar{W}_o\left[1 + \sin\left(\frac{1}{2}\,T\right)\right] \tag{11}$$

where W_O is the applied steady-state load consistent with the eccentricity, $\varepsilon_O = 0.8$. At each time step a new value of $\bar{W}$ was calculated from equation (11). For values of $\bar{M}$ and Ω given in table I and using $\bar{W}$, $\bar{F}_r$, and $\bar{F}_\theta$, equations (9) and (10) were solved for ε, $\dot{\varepsilon}$, φ, and $\dot{\varphi}$ for the next time step. The rest of the procedure is repeated as described.

Variable Rotating Load

This type of loading is interesting since it pertains to an engine bearing. The data used for the analysis refer to the Ruston and Hornsby 6 VEB-X MKIII connecting rod bearing (ref. 5) and is listed in table I. The polar load diagram is represented by figure 2. For the magnitude of load at different positions of crank angle the reader may refer to appendix 4.1 of the paper (ref. 5). The time step $\Delta T = \pi/18$ was taken in this case to match with the

given applied load at 10° crank angle interval. The resultant applied load was nondimensionalized using the expression $\bar{W} = WC^2/\eta\omega R^3L$. This load was introduced in the equations of motion to determine the position of the journal center (ε, $\dot{\varepsilon}$, φ, and $\dot{\varphi}$). The hydrodynamic forces F_r and F_θ were, however, computed after solving equation (4).

RESULTS AND DISCUSSION

To assure ourselves that our formulation is consistent with reference 2, our results are shown in figure 3(a) and compared to those of reference 2 shown in figure 3(b). The comparison indicates very good agreement for the conditions stated in the figure. In figure 4(a) a typical three-dimensional pressure distribution for a particular time step is shown. From this figure one can see the cavitated region and the variation of pressure throughout the complete clearance space of the bearing. Figure 5 gives the journal locus when the bearing is stable under the action of constant unidirectional load. In figure 3, the journal trajectory within the clearance circle is shown for an unstable bearing. The journal locus reaches a limit cycle. This type of observation has been made by many workers (refs. 6 to 8) while dealing with short and infinitely long bearings.

Having compared the result of the present solution with that of Akers et al. (ref. 2), an attempt was made to study the effect of periodic and variable load. Figure 6 shows the journal locus for a periodic load superimposed on the constant load given the same as those of a unidirectional constant load. From these two figures it may be seen that a bearing which is stable under the action of a unidirectional constant load can be made unstable when a periodic load is applied. In the latter case the journal locus reaches a limit cycle.

The journal center trajectory of the connecting rod bearing (variable rotating load) is shown in figure 7. The trajectory is very complex: it neither tends to go to an equilibrium point nor to reach a limit cycle. It may be mentioned that the value of the minimum film thickness for this case exceeds that obtained by others using the mobility method of solution. The mobility method (ref. 3), however, does not consider the stability of the system.

The present solution does not take account of the transport of fluid through the cavitated region and consequently does not take full account of the oil film history. A subsequent analysis has been undertaken that will include the oil film history effects for these dynamic conditions and will be compared at a later date. A proper accounting of the mass flow in time is believed to be important in dynamically loaded bearings and should influence the trajectory of the rotor in a more realistic way.

CONCLUSIONS

From the nonlinear transient analysis of an oil-film journal bearing under different dynamic loads with Reynolds type boundary conditions, the following conclusions are evident.

1. Although the analysis is costly in terms of computer time, it gives the orbital trajectory within the clearance circle which is not obtainable using a simplified (linearized) theory.

2. As it is shown by others while dealing with short and infinitely long bearing theory, the journal locus for a finite bearing ends in a limit cycle for an unstable system.

3. A stable journal with a constant unidirection load can be made unstable by superimposing a periodic load on the system.

4. A more correct (cavitated) boundary condition using the prehistory of film may show some interesting phenomena which are not revealed in the present method of solution.

REFERENCES

1. Allaire, P.E., Design of Journal Bearings for High-Speed Rotating Machinery, Fundamentals of the Design of Fluid Film Bearings, ASME Publication, New York, 1979, pp. 45-83.

2. Akers, A., Michaelson, S., and Cameron, A., Stability Contours for a Whirling Finite Journal Bearing, J. Lub. Tech., Trans. ASME, Series F, vol. 93, no. 1, 1971, pp. 177-190.

3. Booker, J.F., Dynamically Loaded Journal Bearings - Mobility Method of Solution, J. Basic Engr. Trans. ASME, Series D, vol. 187, no. 3, 1965, p. 537.

4. Horsnell, R. and McCallion, H., Prediction of Some Journal Bearing Characteristics Under Static and Dynamic Loading, Proc. Lubrication and Wear Convention, Inst. Mech. Engrs., London, 1963, pp. 126-138.

5. Campbell, J. Love, P.P., Martin, F.A., and Refique, S.C., Bearings for Reciprocating Machinery: A review of the Present State of Theoretical Experiment and Service Knowledge, Conf. Lubrication and Wear, Proc. Inst. Mech. Engrs., vol. 182, (Part 3A), 1967-68, pp. 51-74.

6. Kirk, R. and Guenter, E.J., Transient Journal Bearing Analysis, NASA CR-1549, June 1970.

7. Holmes, R., The Vibration of a Rigid Shaft on Short Sleeve Bearings, J. Mech. Engr. Sci., vol. 2, no. 4, 1960, p. 337.

8. Jakobsen, K. and Christensen, H., Nonlinear Transient Vibrations in Journal Bearings, Proc. Inst. Mech. Engr. vol. 183, Part 3P, 1968-69, pp. 50-56.

TABLE I. - INPUT DATA FOR VARIOUS CONDITIONS OF LOAD

Condition	L/D	ε_0	M	Ω	ω(RAD/S)	D(M)	C/R	η(Pa·s)
Unidirectional constant	1.0	0.8	5.	0.5	-----	---	------	----
Unidirectional periodic	1.0	0.8	5.	0.5	-----	---	------	----
Variable rotating	0.562	0.724	5.	1.0	62.84	0.2	0.0008	0.15

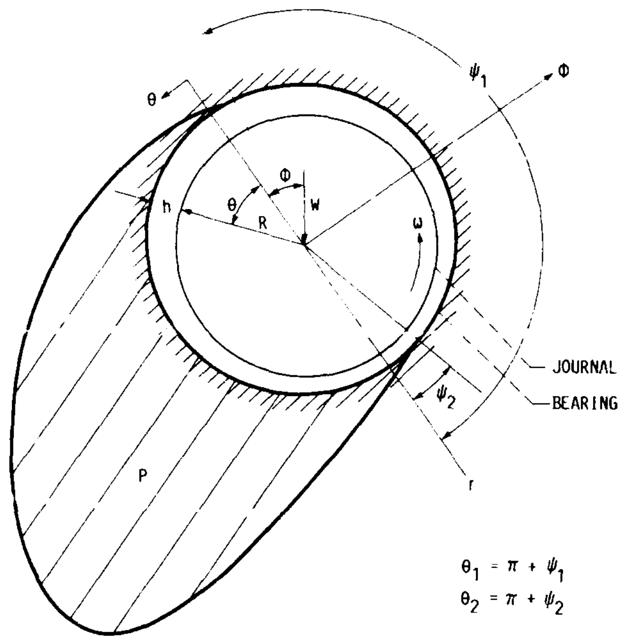


FIGURE 1. - A SCHEMATIC DIAGRAM OF JOURNAL BEARING.

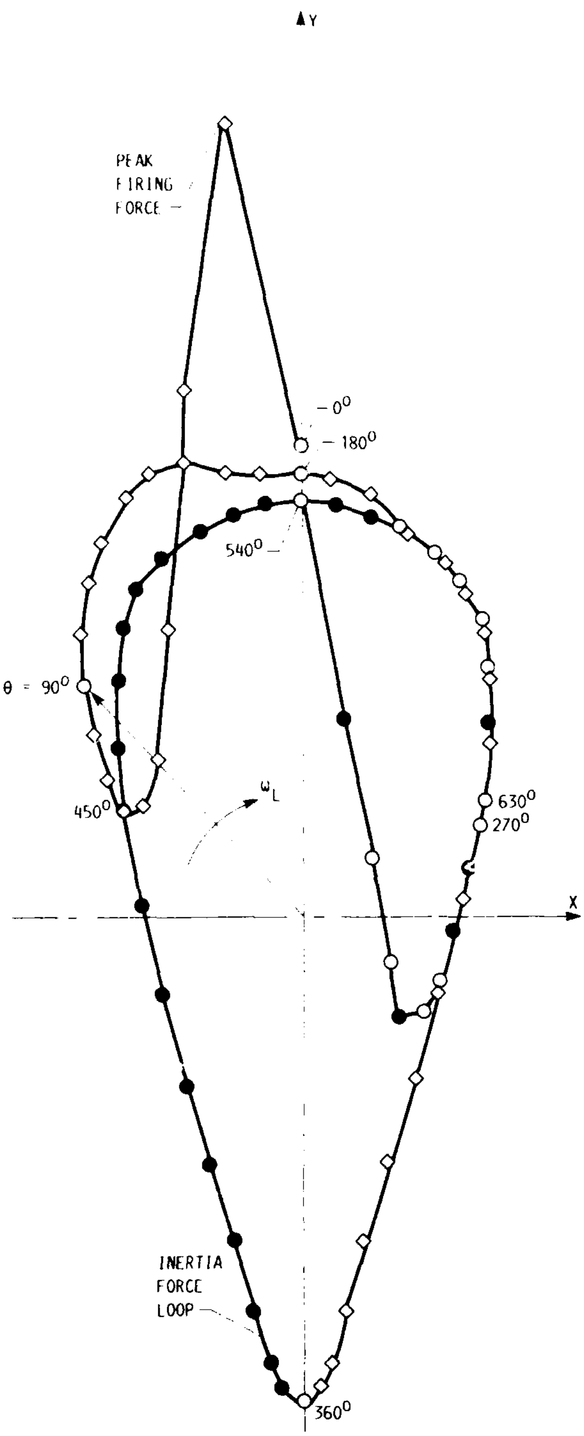
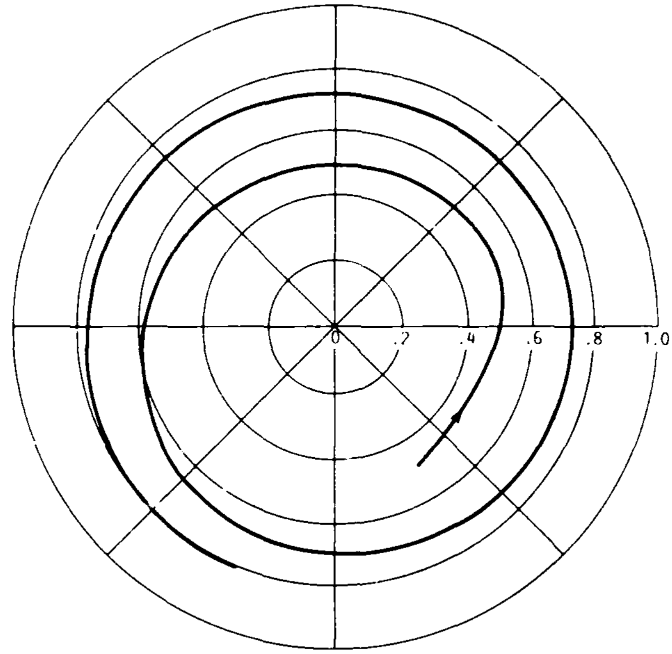
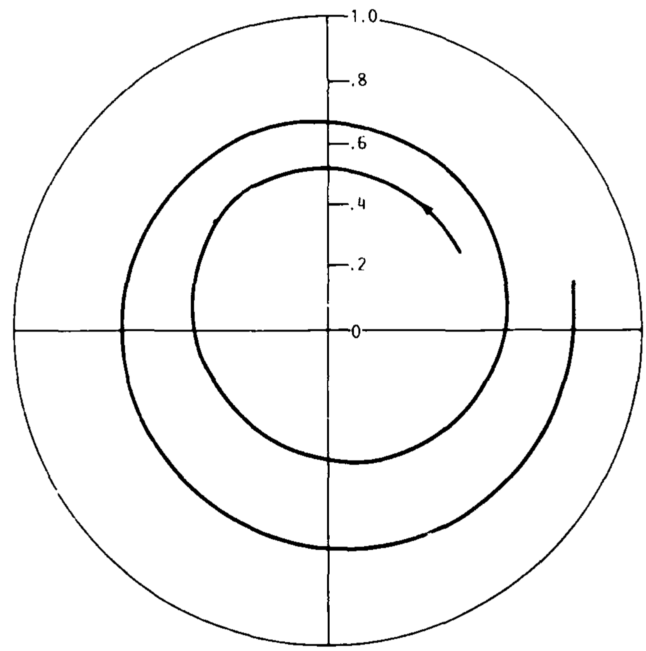


FIGURE 2. - POLAR LOAD DIAGRAM OF ENGINE BEARING.



(A) OBTAINED FROM THE PRESENT METHOD SOLUTION.



(B) OBTAINED BY AKERS ET AL. [2].

FIGURE 3. - JOURNAL CENTER TRAJECTORY. (L/D = 1.0, ε_0 = 0.5, ψ_0 = 30°, $\bar{M}$ = 10.24, Ω = 0.5).

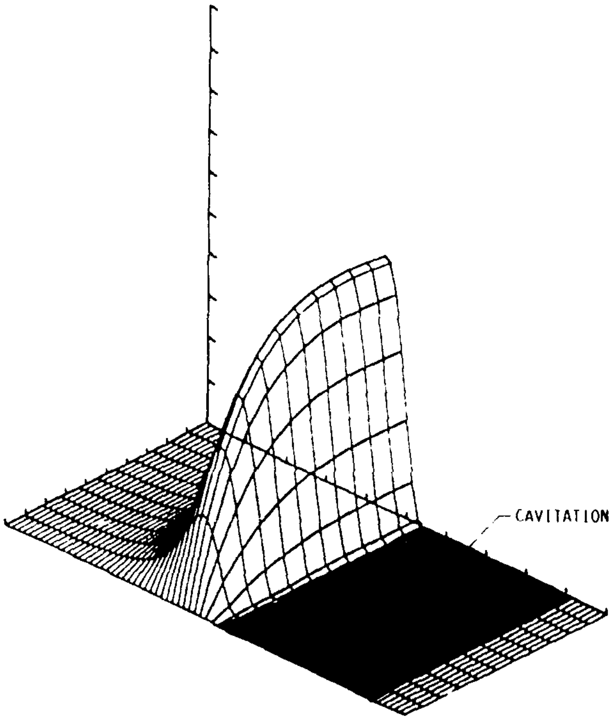


FIGURE 4. A TYPICAL PRESSURE DISTRIBUTION (L/D 1.0, ε_0 = 0.8, $\overline{M}$ 5, Ω 0.5, T 0).

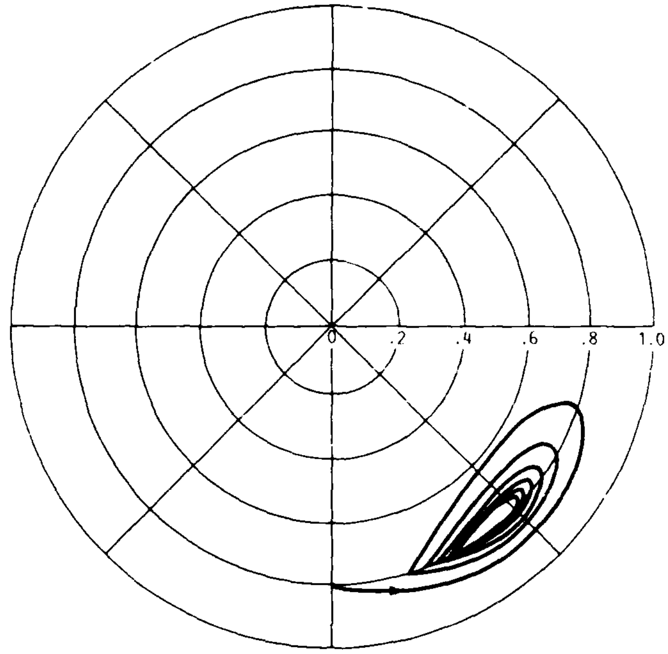


FIGURE 5. - JOURNAL CENTER TRAJECTORY FOR A UNIDIRECTIONAL CONSTANT LOAD (L/D = 1.0, ε_0 = 0.8, $\overline{M}$ = 5, Ω = 0.5).

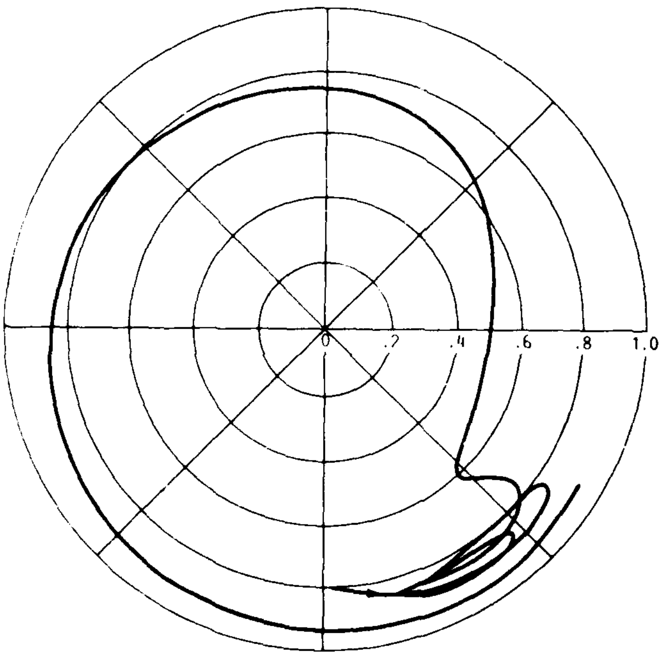


FIGURE 6. - JOURNAL CENTER TRAJECTORY FOR A UNIDIRECTIONAL PERIODIC LOAD (L/D - 1.0, ε_0 - 0.8, $\overline{M}$ = 5, Ω = 0.5).

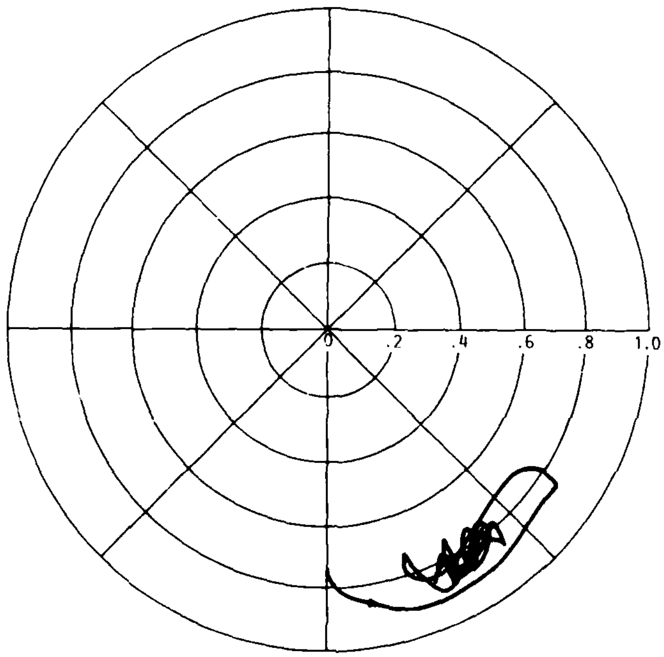


FIGURE 7. - JOURNAL CENTER TRAJECTORY FOR VARIABLE ROTATING LOAD.

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